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Transfer and weighting functions of a sewage treatment plant based on random input and output signal characteristics

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Abstract

The paper is an attempt to describe application of the statistical identification technique for practical problems of the dynamic response of a typical functioning sewage treatment plant. Mathematical model of the plant is based on characteristics of random input and output signals obtained simultaneously. This idea has been applied to the stationary linear dynamic sewage treatment plant whose model is described by the weighting function and the transfer function. The plant weighting function was determined from the autocorrelation function of the input and the cross-correlation function of input and output using integral Wiener-Hopf equation. The transfer function was related with the weighting function of Laplace transformation.

Keywords: Stationary linear dynamic plant; Correlation analysis; Weighting and transfer functions

1. Introduction

Application of statistical methods to experimental data employs the dynamical description of a system by applying statistical methods to experimental data in case of incomplete information about the processes involved or when the processes are to complicated to be described by a deterministic model. The sets of data taken during normal exploitation of a system are used. The problem of identification and process-parameter estimation from statistical measurements is encountered in industrial, biological, agricultural and other processes. Random variations in the input forcing signal along with the sewage treatment plant response form the base for developing a mathematical model by the statistical correlation techniques [1–3,9]. If dynamical movement of a system is limited to a narrow area of normal operation than the construction of a statistic model of a plant is often based on the stationary linear lumped parameters model. This model is unduly crude but simple and easy in practical application. A complex system might be so complicated that even full available statistical information about the state of its elements doesn't make it possible to establish functionality of the system in general. In that case the idea of a black box is introduced, having multiple random inputs and multiple random outputs and the reactions on any external interaction, with unknown internal bonds. There are processes in systems for which the state of the object could not be changed at will (astronomical objects, medical diagnostic, economical analysis) and in that case to analyse these objects the idea of a black box is used. To establish relations between information sets the methods of nonlinear many-dimensional sequential regression is applied by forming a hypersurface of the system reaction on the external interactions. An increase of the dynamical description precision leads to an increase on information use and for a specific closed set of experimental data this type of analysis is too labour-consuming and gives poor practical results. More reasonable seems to apply the statistical averaging estimators for the mathematical description of stochastic signals in effects leading to graphical images of the input autocorrelation function and the output cross-correlation function. For the description of these functions one needs to apply the transformed analytical functions. If the knowledge about the interaction between various system signals is lacking then in a simplified model the matrixes of the correlation functions and dynamical characteristics are reduced to the diagonal matrices. From the solution of a system of the vector-matrix stochastic integral equations including state parameters, random vector input and output, and random vector initial condition, unknown matrices of parameter constants describing mathematical model of a black box could be obtained. The system analysed in this paper may be treated as a black box in case if only the estimation of the municipal sewage treatment

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plant efficiency is needed in regard to environmental protection regulations. However, the knowledge of the system structure and functioning of its elements enables to establish the block structure of the system of control interactions for a real and functioning municipal sewage treatment plant.

2. Description of the sewage treatment plant

The experimental data of indices specifying concentration of five main components in raw and treated sewage were obtained from a typical mechanical–biological sewage treatment plant using municipal sewage from the West-Pomeranian region in the north-west of Poland. A simplified operational schematic diagram of this plant is presented in Fig. 1. The technology of sewage treatment is based on an activated sludge process. Plant installation consists of intermediary and secondary settlement tanks connected with two aeration activated sludge chambers having two separate recirculation systems of activated sludge. The main plant structures are localised in an embankment what causes the gravitational flow of sewage through the plant to the river. Raw sewage flows through the grating in the building and large litter is stopped, removed and transported by conveyour-belt system to the container placed at the base of embankment were it is treated with chlorinated lime and finally transported outside the plant to the communal dumping ground. Near raw sewage collection points of raw sewage the water hydrants are placed for maintaining cleanness in an area of sewage release with street inlets to the local drainage system. After flowing through the grating, sewage is directed to the two-chamber horizontal sand tank. Mineral suspension is removed periodically by a mammoth pump and transported to the plot of ground at the base of sand tank. Mechanically cleaned sewage is directed to the distribution well and further to the activated sludge aeration chambers. To assure the closest contact of sewage with activated sludge, the contents of the chambers with activated sludge is mixed mechanically. Due to the action of aerobic micro-organisms the organic component of sewage as well as biogenic substances are mineralised. The most obvious harmful effect of biodegradable organic matter in sewage is BOD, consisting of a biochemical oxygen demand for dissolved oxygen by microorganism living in an activated sludge. The result of contamination removal of by an activated sludge of biogenic substances is described by the so-called removal effect parameters (N_{tot} and P_{tot}) expressed as the ratio of concentration in comparison to the respective concentrations in untreated sewage. Another method of organic substances removal is the process of their chemical oxidation. This process is described by the COD (chemical oxygen demand) index. For our mechanical–biological sewage treatment plant this index may be used alternatively. The excess of sediments containing non-decomposed input sediments together with biological secondary sediments is removed to the excess sediment concentrators and next to separate fermentation chambers where fermentation process takes place. Water from the excess sediment chambers, fermentation chambers and sedimentation lagoon is collected in a special chamber and directed to the raw sewage collection points. The raw and treated sewage is evaluated on the sediment contents and directed to a natural collector. Concentration measurements for suspension (Polish Standard-72/C-04559), biogenic compounds of nitrogen (Polish Standard-73/C-04576) and phosphorus (Polish Standard-C-04537-14) and organic compounds (BOD, Polish Standard-84/C-04578.05, COD, Polish Standard-74/C-04578/03) were performed according to a common procedure in order to determine the amount of each component. The results of measurements were presented in form of the discrete values of random function input signals as well as output signals obtained simultaneously. The number of measurements for each component was 120.

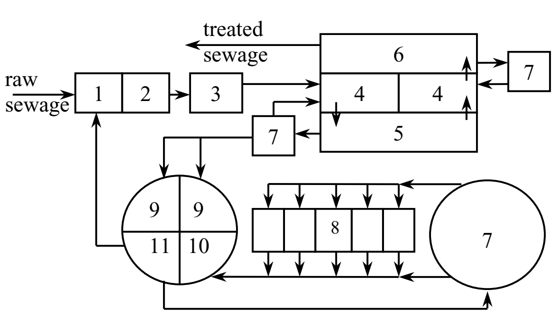

Fig. 1. Block diagram of a functioning municipal sewage treatment plant in West Pomeranian region in Poland. 1: raw sewage tank; 2: grate; 3: two-chamber horizontal sand tank; 4: biological treatment tank; 5: intermediary settlement tank; 6: secondary settlement tank; 7: recirculation of activated sludge; 8: sedimentation lagoon; 9: sludge consolidation tank; 10: tank for cleaned water taken from over the sludge; 11: pumping station.

3. Theoretical description

Statistical methods are often used by many researchers to obtain quickly a rough estimate of the structure and parameters of a mathematical model for a functioning industrial plant. The results should be quite satisfactory and highly effective in plants of varied and complicated physical nature whereas other methods yield no practical results. Very often the dynamic characteristics of an industrial plant are obtained using the correlation analysis [4,5]. When the random vector functions $\bar{\boldsymbol{m}}(t), \bar{\boldsymbol{u}}(t) \in \{\bar{\boldsymbol{v}}(t)\}$ are stationary then the correlation matrix of the input random vectors $\bar{\boldsymbol{m}}(t)$ as well as the cross-correlation matrix of input and output random vectors $\bar{\boldsymbol{u}}(t)$ depend only on the interval $t_2 - t_1 = \tau$ and are independent of the position of these intervals in the range of argument t and τ. Consequently, these matrixes may be found by solving the following integral equation [1,6]:

$$\boldsymbol{R}_{vv}(\tau) = \underset{T\to\infty}{\text{limit}} \frac{1}{T-\tau}\int_0^{T-\tau} [\bar{v}(t+\tau) - \bar{\mu}_v][\bar{v}(t) - \bar{\mu}_v]^T \, dt \tag{1}$$

where $\bar{\mu}_v(i = \overline{1, n})$ is the expectation constant vector of a random vector $\bar{v}(t)$. When the random functions $v_i(t)$ and $v_j(t)$ are ergodic, stationary, and statistically independent, then for a limited time of random vector existence the correlation matrixes are diagonal matrixes and each matrix component is defined by the equation

$$R^*_{vv}(\tau, T) = \frac{1}{T}\int_0^{T} [\bar{v}(t+\tau) - \bar{\mu}_v][\bar{v}(t) - \bar{\mu}_v]^T \, dt \tag{2}$$

As follows from Eq. (2) the accuracy of the correlation function determination depends on the length of integration interval T and by selecting the value of T the random function $v(t)$ should be related to a certain class of random functions having a known structure of the correlation function, i.e. to built a mathematical model. If this model describes the random function sufficiently well than the correlation function will be optimised in the sense of mean square deviation

$$\sigma_R^2(\tau, T) = \frac{2}{T^2}\int_0^{T} (T-\delta)[R_v^2(\delta) + R_v(\tau+\delta)(R_v(\delta-\tau)] \, d\delta \tag{3}$$

In practice only discrete random functions within a limited time interval T are available and in that case instead of (3) yet another criterion limiting calculations of correlation functions to the values of time shift defined by the relation

$$\tau = \tau_{\max} \quad \text{if} \quad |\boldsymbol{R}(\tau)| \le 0.05\, |\boldsymbol{R}(0)| \tag{4}$$

is used.

Eq. (2) to calculate the correlation function by truncating the sum to a finite number of terms has the following form

$$R^*_{v_i v_i}(k) \approx \frac{1}{N-k}\sum_{j=1}^{N-k} \overset{0}{v}_{j_j} \overset{0}{v}_{j+k} \tag{5}$$

where $T/\Delta t = N$ and $\tau \to k$.

Normally the plant-weighting matrix is determined through the convolution integral [3]

$$\bar{\boldsymbol{R}}_{\bar{m}\bar{u}}(\tau) = \int_0^t \boldsymbol{W}(\tau)\boldsymbol{R}_{\bar{m}\bar{m}}(t-\tau) \, d\tau \tag{6}$$

In determination of the weighting matrix $\boldsymbol{W}(\tau)$ by solving the convolution integral (6), the auto-correlation and the cross-correlation matrices should be used as, respectively

$$\boldsymbol{R}_{\bar{m}\bar{m}}(\tau) = \begin{cases} \boldsymbol{R}^+_{\bar{m}\bar{m}}(\tau) & \text{at} \quad \tau \ge 0 \\ \boldsymbol{R}^-_{\bar{m}\bar{m}}(\tau) & \text{at} \quad \tau \le 0 \end{cases} \tag{7}$$

$$\boldsymbol{R}_{\bar{m}\bar{u}}(\tau) = \begin{cases} \boldsymbol{R}^+_{\bar{m}\bar{u}}(\tau) & \text{at} \quad \tau \ge 0 \\ \boldsymbol{R}^-_{\bar{u}\bar{m}}(\tau) & \text{at} \quad \tau \le 0 \end{cases} \tag{8}$$

Thus, the convolution integral can be represented by the following form at $t \ge 0$:

$$\operatorname{diag}\{\boldsymbol{R}^+_{\bar{m}\bar{u}}(\tau) - \boldsymbol{R}^-_{\bar{u}\bar{m}}(\tau)\} = \int_0^t \operatorname{diag}\{\boldsymbol{W}(\tau)\} \times \operatorname{diag}\{\boldsymbol{R}^+_{\bar{m}\bar{m}}(t-\tau) - \boldsymbol{R}^-_{\bar{m}\bar{m}}(t-\tau)\}\, d\tau \tag{9}$$

where $\bar{m}(t)$ and $\bar{u}(t)$ are $\{c_{sus}(t); BOD(t); COD(t); c_N(t); c_P(t)\}$ as inlet and outlet stochastic signals.

Describing each element of these correlation matrices in time domain by analytical functions and if for these functions Laplace transforms [7] exist, than. the solution of Eq. (9) gives the transfer functions matrix [8]

$$\boldsymbol{W}(s) = \frac{\boldsymbol{S}^+_{\bar{u}\bar{m}} - \boldsymbol{S}^-_{\bar{m}\bar{u}}(s)}{\boldsymbol{S}^+_{\bar{m}\bar{m}}(s) - \boldsymbol{S}^-_{\bar{m}\bar{m}}(s)} \tag{10}$$

Taking the inverse Laplace transform of the transfer function (10) we obtain the plant weighting function matrix

$$\boldsymbol{W}(t) = \frac{1}{2\pi j}\int_{\alpha-j\infty}^{\alpha+j\infty} \left[\frac{\boldsymbol{S}^+_{\bar{u}\bar{m}}(s) - \boldsymbol{S}^-_{\bar{m}\bar{u}}(s)}{\boldsymbol{S}^+_{\bar{m}\bar{m}}(s) - \boldsymbol{S}^-_{\bar{m}\bar{m}}(s)}\right] e^{st} \, ds \tag{11}$$

In order to find the solution of Eq. (10) it is necessary to describe each component of the transfer functions matrix as a composition of both a numerator and a denominator. Then the time response for each measured component of sewage can be obtained by use of the residuum methods.

4. Results of calculation

In this paper a set of 120 data points for five main components of a raw sewage and of treated sewage which were measured synchronously for a specific time interval in a normally functioning treatment sewage plant are used. Static characteristics and the variation range and allowed values of analysed indicators for untreated and treated sewage are presented in Table 1. The results of measurements are presented in form of discrete stochastic functions. The character of these functions for analysed indicators is similar and as an example we present in Figs. 2 and 3 the time variation of BOD indicator for raw and treated sewage. Analytical description has been based on estimators defined in Eq. (5) which was applied to calculate the values of the autocorrelation functions and the cross-correlation functions. As an example, the results of these calculations for phosphorus are presented in Figs. 4 and 5. They show the variation of the correlation functions with time for the raw and the treated sewage. The points in these figures represent the values of the correlation function estimators. Similar figures were made for other used indicators but they are not published here.

It is assumed that generally the structures of the correlation functions of random input and output signals are represented by an analytical expression having Laplace transform. This transform gives well defined relationships between the time domain and the frequency domain description. Analysis of the correlation functions presented in Figs. 4 and 5 for phosphorus and other measured indicators lead us to propose the same analytical form of the auto-correlation (12) and

Table 1
Statistical characteristics, limiting values and admissible values for five main indicators for raw and cleaned sewage

$\bar{V}$	Raw sewage				Treated sewage				Admissible values[a]
	$E(v)$	$\Delta(v)$	$\sigma(v)$	$v_{min} - v_{max}$	$E(v)$	$\Delta(v)$	$\sigma(v)$	$v_{min} - v_{max}$	
BOD	251.70	34.76	46.93	160–360	10.23	1.78	2.06	6–15	30.0 mgO_2/l
COD	488.56	50.57	65.78	458–693	49.72	5.92	7.09	34–65	150.0 mgO_2/l
Suspension	123.75	14.26	17.00	88–163	11.21	2.81	3.4	4-22	50.0 mg/l
N_{tot}	121.42	17.37	20.33	76–163	8.35	1.79	2.11	4.3-13.2	30.0 mgN/l
P_{tot}	11.29	0.93	1.15	9–14.5	1.24	0.16	0.20	0.91–2.1	5.0 mgP/l

[a] RP Parlament Law Register (Poland) No. 79, 503, 5 November 1991.

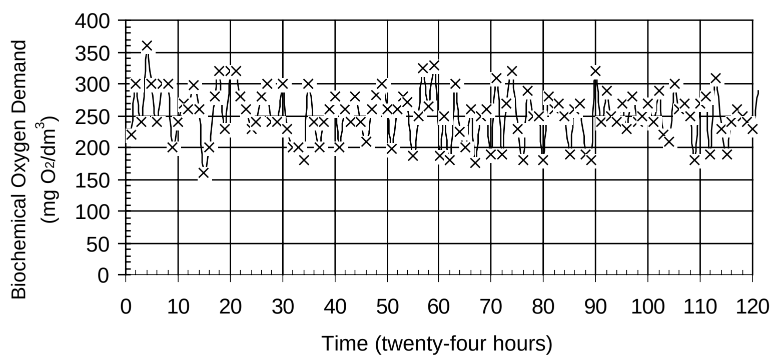

Fig. 2. Temporal change of BOD indicator for raw sewage.

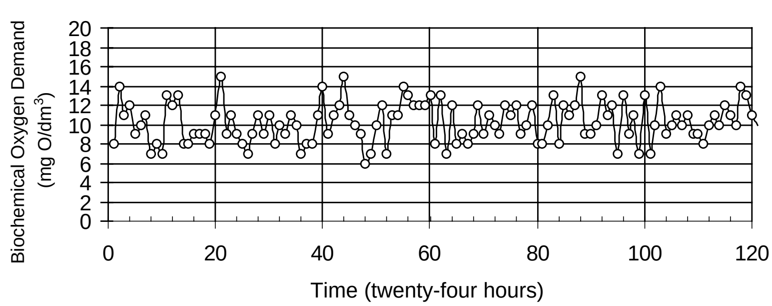

Fig. 3. Temporal change of BOD indicator for treated sewage.

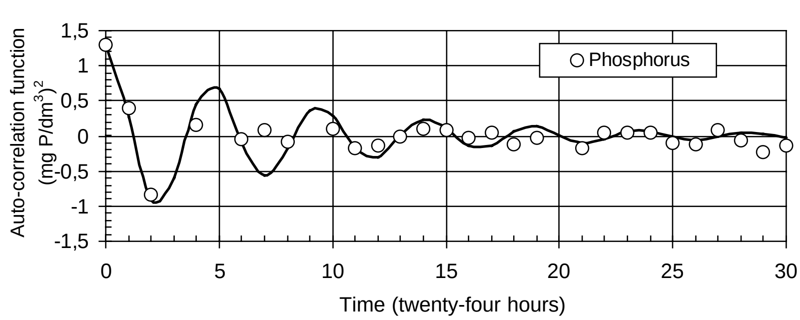

Fig. 4. Auto-correlation function for P_{tot} for raw sewage.

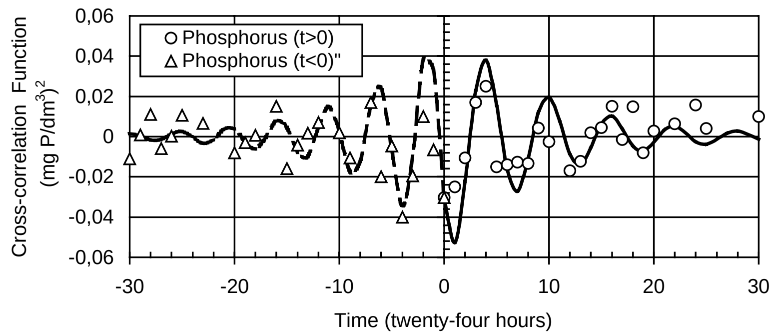

Fig. 5. Cross-correlation function for P_{tot}.

cross-correlation (13) functions for the investigated sewage components [7].

$$\boldsymbol{R}_{\bar{m}\bar{m}}(\tau) = \boldsymbol{A}\mathrm{e}^{-\alpha|\tau|}\cos(\boldsymbol{\omega}\tau) \tag{12}$$

$$\boldsymbol{R}^{+}_{\bar{u}\bar{m}}(\tau) - \boldsymbol{R}^{-}_{\bar{m}\bar{u}}(\tau) = [\boldsymbol{B}\cos(\boldsymbol{\gamma}\tau) + \boldsymbol{D}\sin(\boldsymbol{\gamma}\tau)]\mathrm{e}^{-\boldsymbol{\beta}\tau} - [\boldsymbol{B}\cos(\boldsymbol{\omega}\tau) + \boldsymbol{C}\sin(\boldsymbol{\omega}\tau)]\mathrm{e}^{\boldsymbol{\alpha}\tau} \tag{13}$$

The functions have the following Laplace transforms in a frequency domain [7]:

$$\boldsymbol{S}^{+}_{\bar{m}\bar{m}}(s) - \boldsymbol{S}^{-}_{\bar{m}\bar{m}}(s) = \boldsymbol{A}\left\{\frac{s+\boldsymbol{\alpha}}{(s+\boldsymbol{\alpha})^2+\boldsymbol{\omega}^2} - \frac{s-\boldsymbol{\alpha}}{(s-\boldsymbol{\alpha})^2+\boldsymbol{\omega}^2}\right\} \tag{14}$$

$$\boldsymbol{S}^{+}_{\bar{u}\bar{m}}(s) - \boldsymbol{S}^{-}_{\bar{m}\bar{u}}(s) = \left\{\boldsymbol{B}\frac{s+\boldsymbol{\beta}}{(s+\boldsymbol{\beta})^2+\boldsymbol{\gamma}^2} + \boldsymbol{D}\frac{\boldsymbol{\gamma}}{(s+\boldsymbol{\beta})^2+\boldsymbol{\gamma}^2}\right\} - \left\{\boldsymbol{B}\frac{s-\boldsymbol{\alpha}}{(s-\boldsymbol{\alpha})^2+\boldsymbol{\omega}^2} + \boldsymbol{C}\frac{\boldsymbol{\omega}}{(s-\boldsymbol{\alpha})^2+\boldsymbol{\omega}^2}\right\} \tag{15}$$

The values of constants appearing in these equations for five sewage components are given in Tables 2 and 3. Inserting the above transforms into Eq. (11) and after some simple algebraic manipulations the following general form of the transfer function could be obtained

$$\boldsymbol{W}(s) = \frac{1}{2\boldsymbol{\alpha}A}\left[\boldsymbol{F}_0 + \frac{\boldsymbol{F}_1}{s+\sqrt{(\boldsymbol{\alpha}^2+\boldsymbol{\omega}^2)}} + \frac{\boldsymbol{F}_2}{s-\sqrt{(\boldsymbol{\alpha}^2+\boldsymbol{\omega}^2)}} + \frac{\boldsymbol{F}_3 s + \boldsymbol{F}_4}{(s+\boldsymbol{\beta})^2+\boldsymbol{\gamma}^2}\right] \tag{16}$$

where

$$\boldsymbol{F}_0 = \boldsymbol{B\alpha} + \boldsymbol{B\beta} + \boldsymbol{C\omega} - \boldsymbol{D\gamma}$$

Table 2
Parameters of the auto-correlation function of the input random signals

$\bar{V}$	$\boldsymbol{A}$	$\boldsymbol{\alpha}$	$\boldsymbol{\omega}$
Suspension	286.75	0.076	1.22
BOD_5	1685.42	0.056	0.77
COD	4291.65	0.15	1.13
N_{tot}	410.14	0.10	0.88
P_{tot}	1.30	0.12	1.33

$$\boldsymbol{F}_1 = \frac{\begin{array}{l}\{\boldsymbol{F}_0(\boldsymbol{\alpha}^2+\boldsymbol{\omega}^2)(\boldsymbol{\alpha}-\sqrt{\boldsymbol{\alpha}^2+\boldsymbol{\omega}^2})\\ +[\boldsymbol{B}(\boldsymbol{\beta}^2+\boldsymbol{\gamma}^2-\boldsymbol{\alpha}^2-\boldsymbol{\omega}^2)\\ +2\boldsymbol{\alpha D\gamma}+2\boldsymbol{\beta C\omega}](\boldsymbol{\alpha}^2+\boldsymbol{\omega}^2+\boldsymbol{\alpha}\sqrt{\boldsymbol{\alpha}^2+\boldsymbol{\omega}^2})\\ -[(\boldsymbol{\alpha}^2+\boldsymbol{\omega}^2)(\boldsymbol{B\beta}+\boldsymbol{D\gamma})+(\boldsymbol{\beta}^2+\boldsymbol{\gamma}^2)\\ \times(\boldsymbol{B\alpha}-\boldsymbol{C\omega})](\boldsymbol{\alpha}-\sqrt{\boldsymbol{\alpha}^2+\boldsymbol{\omega}^2})\}\end{array}}{(\boldsymbol{\beta}-\sqrt{\boldsymbol{\alpha}^2+\boldsymbol{\omega}^2})^2+\boldsymbol{\gamma}^2)}$$

$$\boldsymbol{F}_2 = \frac{\begin{array}{l}\{\boldsymbol{F}_0(\boldsymbol{\alpha}^2+\boldsymbol{\omega}^2)(\boldsymbol{\alpha}+\sqrt{\boldsymbol{\alpha}^2+\boldsymbol{\omega}^2})\\ +[\boldsymbol{B}(\boldsymbol{\beta}^2+\boldsymbol{\gamma}^2-\boldsymbol{\alpha}^2-\boldsymbol{\omega}^2)+2\boldsymbol{\alpha D\gamma}+2\boldsymbol{\beta C\omega}]\\ \times(\boldsymbol{\alpha}^2+\boldsymbol{\omega}^2+\boldsymbol{\alpha}\sqrt{\boldsymbol{\alpha}^2+\boldsymbol{\omega}^2})\\ -[(\boldsymbol{\alpha}^2+\boldsymbol{\omega}^2)(\boldsymbol{B\beta}+\boldsymbol{D\gamma})+(\boldsymbol{\beta}^2+\boldsymbol{\gamma}^2)(\boldsymbol{B\alpha}-\boldsymbol{C\omega})]\\ \times(\boldsymbol{\alpha}+\sqrt{\boldsymbol{\alpha}^2+\boldsymbol{\omega}^2})\}\end{array}}{(\boldsymbol{\beta}+\sqrt{\boldsymbol{\alpha}^2+\boldsymbol{\omega}^2})^2+\boldsymbol{\gamma}^2}$$

$$\begin{aligned}\boldsymbol{F}_3 &= 2(\boldsymbol{\alpha}-\boldsymbol{\beta})\boldsymbol{F}_0 + \boldsymbol{B}(\boldsymbol{\beta}^2+\boldsymbol{\gamma}^2-\boldsymbol{\alpha}^2-\boldsymbol{\omega}^2) + 2\boldsymbol{\alpha D\gamma}\\ &\quad + 2\boldsymbol{\beta C\omega} - (\boldsymbol{F}_1+\boldsymbol{F}_2)\boldsymbol{F}_4 = (\boldsymbol{\alpha}^2+\boldsymbol{\omega}^2)(\boldsymbol{B\beta}+\boldsymbol{D\gamma})\\ &\quad + (\boldsymbol{\beta}^2+\boldsymbol{\gamma}^2)(\boldsymbol{B\alpha}-\boldsymbol{C\omega}) - \boldsymbol{F}_0(\boldsymbol{\beta}^2+\boldsymbol{\gamma}^2)\\ &\quad - \frac{(\boldsymbol{F}_1-\boldsymbol{F}_2)(\boldsymbol{\beta}^2+\boldsymbol{\gamma}^2)}{\sqrt{\boldsymbol{\alpha}^2+\boldsymbol{\omega}^2}}\end{aligned}$$

The denominator of the transfer function (16) has a positive real root ($s = +\sqrt{\boldsymbol{\alpha}^2+\boldsymbol{\omega}^2}$) causing an unstable impulse characteristic. The existence of a stable solution of Eq. (16) requires that the term $\boldsymbol{F}_2$ in Eq. (16) must be eliminated [4]. Equating to zero the term $\boldsymbol{F}_2$, the dependence of $\boldsymbol{D}$ on other parameters in this term is obtained. Then the Eq. (16) is reduced to the following form:

$$\boldsymbol{W}(s) = \frac{\boldsymbol{b}_3 s^3 + \boldsymbol{b}_2 s^2 + \boldsymbol{b}_1 s + \boldsymbol{b}_0}{\boldsymbol{a}_3 s^3 + \boldsymbol{a}_2 s^2 + \boldsymbol{a}_1 s + \boldsymbol{a}_0} \tag{17}$$

The values of matrix elements of the transfer function for five analysed indicators of the sewage are presented in Table 4.

To determine the constants of the correlation functions (12) and (13) the method of the non-linear estimation was used.

Taking the inverse Laplace transform [7] of the transfer function (17) we obtain the plant weighting function (impulse response) matrix (18).

Table 3
Parameters of the cross-correlation function of the output random signals

$\bar{V}$	$\boldsymbol{B}$	$\boldsymbol{C}$	$\boldsymbol{D}$	$\boldsymbol{\alpha}$	$\boldsymbol{\beta}$	$\boldsymbol{\gamma}$	$\boldsymbol{\omega}$
Suspension	1.37	9.69	11.08	0.076	0.066	0.93	1.22
BOD_5	4.82	11.98	13.18	0.056	0.047	0.64	0.77
COD	52.09	145.54	176.64	0.15	0.093	0.97	1.13
N_{tot}	−1.16	−14.33	−18.96	0.1	0.12	0.73	0.88
P_{tot}	−0.031	−0.046	−0.05	0.12	0.11	1.05	1.33

Table 4
The values of matrix elements of the transfer function for five analysed indicators of the sewage

$\bar{V}$	$\boldsymbol{F}_0$	$\boldsymbol{F}_5$	$\boldsymbol{F}_6$	$\boldsymbol{F}_7$
Suspension	1.712	−8.229	10.534	−4.358
BOD_5	1.286	−3.432	4.379	0.209
COD	5.777	−85.24	149.67	10.109
N_{tot}	0.975	4.722	−10.28	−3.359
P_{tot}	−0.016	0.060	−0.065	0.026

$$\boldsymbol{W}(t) = \frac{1}{2\pi \boldsymbol{A}}\{\boldsymbol{F}_0\delta(t) + \boldsymbol{F}_5 e^{-(\sqrt{\alpha^2+\omega^2})t} + [(\boldsymbol{F}_6\cos(\gamma t) + \boldsymbol{F}_7\sin(\gamma t)]e^{-\beta t}\} \qquad (18)$$

where

$$\boldsymbol{F}_5 = \frac{2(\alpha - \sqrt{\alpha^2+\omega^2})[\boldsymbol{F}_0(\alpha^2+\omega^2) + (\alpha^2+\omega^2)(\boldsymbol{B}\beta + \boldsymbol{D}\gamma) + (\beta^2+\gamma^2)(\boldsymbol{B}\alpha - \boldsymbol{C}\omega)]}{(\beta - \sqrt{\alpha^2+\omega^2})^2 + \gamma^2},$$

$$\boldsymbol{F}_6 = \boldsymbol{B}(\beta^2+\gamma^2-\alpha^2-\omega^2) + 2\alpha \boldsymbol{D}\gamma + 2\beta \boldsymbol{C}\omega + 2(\alpha-\beta)\boldsymbol{F}_0 - \boldsymbol{F}_5,$$

$$\boldsymbol{F}_7 = \frac{1}{\gamma}\left[\frac{\beta^2+\gamma^2}{\sqrt{\alpha^2+\omega^2}}\boldsymbol{F}_5 - \boldsymbol{B}(\beta^2+\gamma^2-\alpha^2-\omega^2) + 2\alpha \boldsymbol{D}\gamma + 2\beta \boldsymbol{C}\omega - \boldsymbol{F}_0(\beta^2+\gamma^2) - \boldsymbol{F}_6\beta\right]$$

As an example, Fig. 6 presents the impulse characteristic for nitrogen. Similar characteristics for other indicators have been calculated but are not shown in this paper. Instead, in Table 4 the values of $\boldsymbol{F}_0$, $\boldsymbol{F}_5$, $\boldsymbol{F}_6$ and $\boldsymbol{F}_7$ appearing in Eq. (18) for five measured indicators are given.

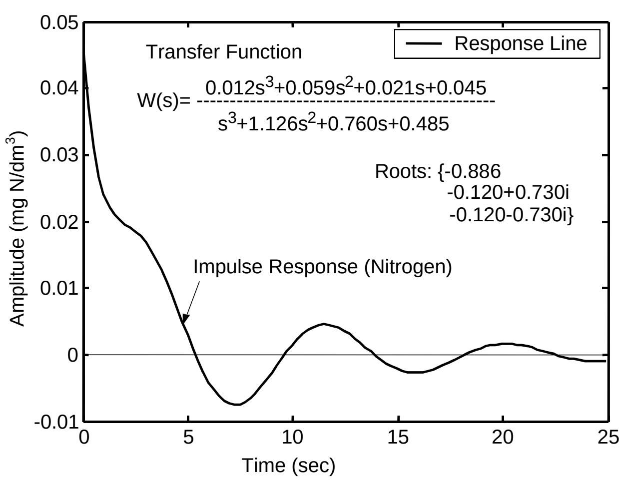

Fig. 6. Impulse response for N_{tot}.

The transfer function (17) is an algebraic image of the linear differential equation for the mathematical model of a system in a dynamical state. Using the property of Laplace transform connecting the derivatives with respect to time and frequency the following form of the differential equation describing the mathematical model of municipal waste treatment plant is obtained

$$a_3\frac{d^3\boldsymbol{R}_{um}(\tau)}{d\tau^3} + \boldsymbol{a}_2\frac{d^2\boldsymbol{R}_{um}(\tau)}{d\tau^2} + \boldsymbol{a}_1\frac{d\boldsymbol{R}_{um}(\tau)}{d\tau} + \boldsymbol{a}_0\boldsymbol{R}_{um}(\tau) = \boldsymbol{b}_3\frac{d^3\boldsymbol{R}_{mm}(\tau)}{d\tau^3} + \boldsymbol{b}_2\frac{d^2\boldsymbol{R}_{mm}(\tau)}{d\tau^3} + \boldsymbol{b}_1\frac{d\boldsymbol{R}_{mm}(\tau)}{d\tau} - b_0\boldsymbol{R}_{mm}(\tau) \qquad (19)$$

where

$$\boldsymbol{a}_3 = 1$$

$$\boldsymbol{a}_2 = (2\beta + \sqrt{\alpha^2+\omega^2})$$

$$\boldsymbol{a}_1 = [(\beta + \sqrt{\alpha^2+\omega^2}) + \gamma^2]$$

$$\boldsymbol{a}_0 = [(\beta^2+\gamma^2)\sqrt{\alpha^2+\omega^2}]$$

$$\boldsymbol{b}_3 = \frac{1}{2\alpha \boldsymbol{A}}(\boldsymbol{B}\alpha + \boldsymbol{B}\beta + \boldsymbol{C}\omega - \boldsymbol{D}\gamma)$$

$$\boldsymbol{b}_2 = \frac{1}{\boldsymbol{A}}(\boldsymbol{B}\alpha + \boldsymbol{B}\beta + \boldsymbol{C}\omega - \boldsymbol{D}\gamma) - \frac{(\alpha^2+\omega^2)(\boldsymbol{B}\beta + \boldsymbol{D}\gamma) + (\beta^2+\gamma^2)(\boldsymbol{B}\alpha - \boldsymbol{C}\omega)}{\sqrt{\alpha^2+\omega^2}}$$

$$\boldsymbol{b}_1 = \left[\frac{1}{2\alpha \boldsymbol{A}}(\boldsymbol{B}\alpha + \boldsymbol{B}\beta + \boldsymbol{C}\omega - \boldsymbol{D}\gamma)(\alpha^2+\omega^2) - \frac{2(\alpha^2+\omega^2)(\boldsymbol{B}\beta + \boldsymbol{D}\gamma) + (\beta^2+\gamma^2)(\boldsymbol{B}\alpha - \boldsymbol{C}\omega)\alpha}{\sqrt{\alpha^2+\omega^2}}\right]$$

$$\boldsymbol{b}_0 = (\alpha^2+\omega^2)(\boldsymbol{B}\beta + \boldsymbol{D}\gamma) + (\beta^2+\gamma^2)(\boldsymbol{B}\alpha - \boldsymbol{C}\omega)\sqrt{\alpha^2+\omega^2})$$

The values for constants appearing in Eq. (19) are given in Table 5. This model takes into account the interdependence

Table 5
The values of constants appearing in Eq. (19)

$\bar{V}$	$\boldsymbol{a}_3$	$\boldsymbol{a}_2$	$\boldsymbol{a}_1$	$\boldsymbol{a}_0$	$\boldsymbol{b}_3$	$\boldsymbol{b}_2$	$\boldsymbol{b}_1$	$\boldsymbol{b}_0$
Suspension	1	1.354	1.031	1.063	0.0393	−0.0944	0.0434	−0.150
BOD_5	1	0.866	0.484	0.318	0.0068	−0.0094	0.0029	−0.0060
COD	1	1.326	1.162	1.082	0.0045	−0.0536	−0.0106	−0.0713
N_{tot}	1	1.126	0.760	0.485	0.012	0.059	0.021	0.045
P_{tot}	1	1.555	1.408	1.488	−0.051	0.073	−0.070	0.153

of five main indicators for the raw and treated sewage, statistically averaged with the help of the correlation functions being the stochastic functions of time.

5. Conclusions

In this article we have attempted to illustrate how the statistical correlation analysis of a developed model can be applied to a real sewage treatment plant. A significant omission in the literature is the lack of any references to the stochastic nature of a real process of this type [10,11]. Thus, our paper may be helpful in description of a sewage treatment plant by a model considering full information with the input and output signal vectors for each component of the system including their self-interaction. The proposed method is simple, effective and accessible for a wide range of industrial users because it employs standard mathematical methods and allows to describe the system by the deterministic statistically averaged mathematical model with dynamical characteristics in time and frequency domain. The method allows also to construct a rough block structure that is needed for the synthesis and analysis of a real municipal sewage treatment plant. However, our approach requires, from the experimental point of view, the execution of a large number of arduous measurements during the length of the process. This may be an obstacle because usually there is a lack of additional measuring points due to the economy.

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